TWICE FLOWN THREE-AXIS MAGNETOMETER EXPERIMENT.

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A three-axis magnetometer experiment was flown onboard the rocket Skøvnung in the summer of 1991 at Skillingaryd (Sweden), and again in the autumn of 1992 at Oldenbroek (The Netherlands). Interesting data were recovered at both flights. The main argument for flying this experiment is its potential for providing very accurate trajectory information. However extracting the trajectory information from magnetometer data is no simple quest.

A method of determining the primary trajectory information (eg. burnout velocity and drag coefficient) from an ideal magnetometer is presented, as are methods for overcoming some common real-world deviations from the ideal case. Measurements from the two flights are presented and compared.

EXTRACTING BASIC FLIGHT PARAMETERS FROM AN IDEAL MAGNETOMETER:

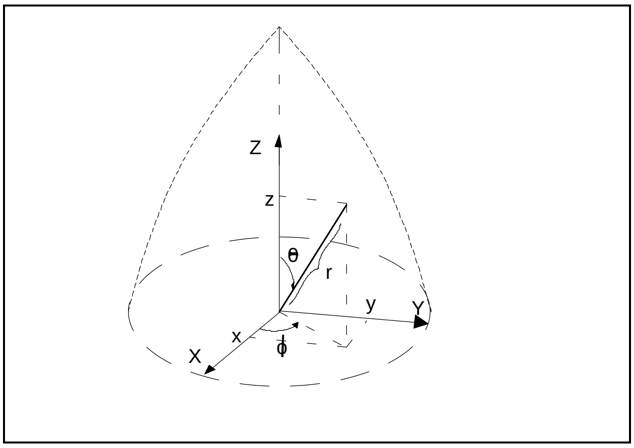
Measuring the Earth's magnetic field in three known orthogonal directions is equivalent of measuring the magnetic field vector, which is known to be constant in both magnitude and direction from a few meters above the surface of the Earth up to very high altitudes. When the system of co-ordinates is moving, the measured direction of the magnetic field vector can be viewed as a way of tracking the measuring system of co-ordinates from a reference viewpoint on the ground. The measuring system of co-ordinates is defined as having its origin at the rockets center of gravity, having the X- and Y-axis at some nonimportant orthogonal radial directions, and the Z-axis pointing in the direction of the nose. The three composants of the magnetic field vector are measured simultaneous and stored in digital memory at time = $n*\Delta T$, where ΔT is the sampling interval and n is the time parameter.

The measured coordinates of the magnetic field are converted into spherical coordinates giving 1: the magnitude r of the magnetic field vector, 2: the angle θ between the magnetic field vector and the roll axis of the rocket, and 3: the angle ϕ between the magnetic field vector and some fixed pitch axis of the rocket. The conversion from the cartesian coordinates x,y,z to the spherical coordinates r,θ,ϕ is made from the expressions:

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\phi = \begin{cases} \arctan(\frac{y}{x}) & if\, x > 0 \\ \pi + \arctan(\frac{y}{x}) & if\, x < 0 \\ \frac{\pi}{2} & if\, x = 0 \end{cases}$$

$$\theta = \arccos(\frac{z}{y}) + \theta_0$$



Orientation of the X, Y, and Z axis with respect to the rocket, and the corresponding spherical representation.

The time record of r(n) makes it possible to confirm, that the measurements are valid by checking that r(t) ≈ 49.5 µT for the complete time record. When data are valid, the angle ϕ(t) tells the complete history of roll, and θ(n) tells the complete history of pitch.

The roll angle itself is not very important, but it can be transformed into the instantaneous roll frequency by 'differentiating' ϕ(n):

$$f_{roll}(n\Delta T) \approx \frac{\phi(n) - \phi(n-1)}{2\pi\Delta T}$$

($\Delta T = 1/f_{sample}$) is the sampling interval. The instantaneous roll frequency should be roughly proportional to the squared speed of the rocket, forcing f_{roll} to have a maximum at burnout.

The most interesting parameter to observe is θ(n), which has to be offset by some θ_0, to make θ(0) equal the launch elevation, which has to be measured very carefully. Alternatively, the launch elevation could be recovered from flight data. In practice the launching ramp is often made from steel, which disturbs the earliest values of θ(n), that has to be abandoned. In most cases however, the burnout elevation roughly equals the launch elevation and may be used for determining θ_0, especially as the final results are only slightly sensitive to the chosen value of θ_0. The best solution is of course doing both, thus gathering redundant information.

When θ is known, in principle the same values could be calculated from a trajectory calculation. As the rockets thrust curve is not exactly known, and the earliest values of θ are not valid, it makes no sense to investigate rocket motion in the powered phase of flight. If the burnout velocity $v_{burnout}$, the drag coefficient c_d, and the burnout weight m are known, the ballistic trajectory can be calculated from the equations of motion:

$$\dot{v}_{xe} = -\frac{1}{2} A \rho c_d v \cdot \frac{v_{xe}}{m} \quad and \quad \dot{v}_{ye} = -\frac{1}{2} A \rho c_d v \cdot \frac{v_{ye}}{m} - g$$

Where A is the reference area, ρ is the air density, g is the gravity, and v, v_{xe} and v_{ye} are the magnitude and the horizontal and vertical composants of the instantaneous velocity in the reference (earth) system of coordinates. Note that c_d and ρ can only be treated as constants for limited values of speed and altitude.

Calculating v_{xe} and v_{ye} provides the speed v of the rocket and the theoretical value Ω of the 'direction of flight' θ:

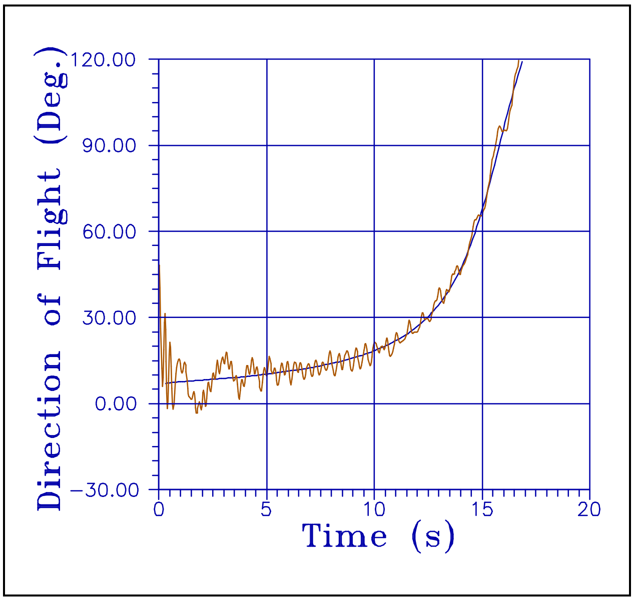
$$v = \sqrt{v_{xe}^2 + v_{ye}^2}$$

$$\Omega = 90^0 - \arccos(\frac{v_{xe}}{v})$$

To summarize: the direction of the magnetic field vector seen from the rocket is equivalent to measuring the attitude of the rocket seen from the ground. This attitude is now to be compared with the theoretical value from a numerical solution of the equations of motion. The value of Ω is now calculated throughout the flight at any time where θ is known. In principle, the basic parameters $v_{burnout}$, c_d, and m are needed for this trajectory calculation, but as they are not known they are estimated by minimizing the index of performance:

$$I_N = \sum_N w(n) \cdot (\bar{\ }(n) - \Omega(n))^2$$

Where w(n) is some suitable non-negative weigthing function, used to suppress unreliable data. Under normal conditions, w(n) = 1 should be chosen. The index 'N' marks any sample from burnout to the end of the ballistic flight phase. The target of this procedure is to determine the trajectory parameters that makes the optimum fit between Ω(n) and θ(n) throughout the ballistic flight phase. A gradient search method can be applied to find the best fit, but the initial guesswork on determining $v_{burnout}$, c_d, and m should be done manually, which can be rather time consuming. When burnout conditions are known, the complete altitude profile is easily calculated by integration of the velocity profile.



The best fit values of Ω(n) compared to the measured values of θ(n). Data from the powered and ballistic flight phases, Skillingaryd 1991.

Applying this method on real flight data (Skøvnung, Skillingaryd/Sweden, July 91') concludes the following: Dragcoefficient = 0.3 referring to the cross sectional area, burnout velocity = 149.5 m/s. Maximum altitude was calculated to 1171 m. The graph shows the values of Ω(n) from the trajectory calculation (smooth curve) with these parameters and the measured values of θ(n) for comparison.

CORRECTING A NON-IDEAL MAGNETOMETER:

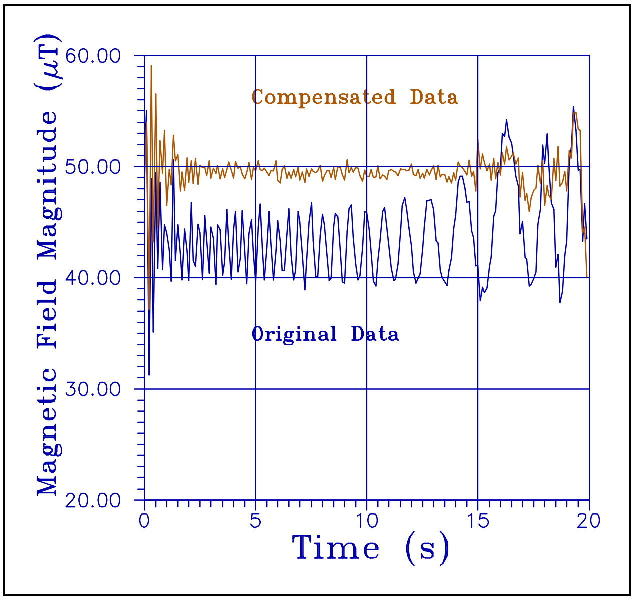
The results from Sweden were processed by using the raw data only. The values of r(n) were oscillating around 50 µT with some correlation to the roll frequency. The reason for this was later known to be magnetometer imperfectness. The primary source of errors is of course, that the sensitivity of the three channels is to be exactly the same, but it turned out that also misalignment of the sensors will disturb the measurements. Fortunately these errors are relatively easy to overcome by 'twisting' the coordinate system. The twisting is done by replacing the original samples x(n), y(n) and z(n) by $x_0(n)$, $y_0(n)$ and $z_0(n)$, where:

$$\begin{pmatrix} x_0(n) \\ y_0(n) \\ z_0(n) \end{pmatrix} = \begin{pmatrix} 1+a_{11} & a_{12} & a_{13} \\ a_{21} & 1+a_{22} & a_{23} \\ a_{31} & a_{32} & 1+a_{33} \end{pmatrix} \cdot \begin{pmatrix} x(n) \\ y(n) \\ z(n) \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

The parameters a_{xx} and b_x are unknown and should initially be set to 0, and are thereafter to be determined by minimizing yet another index of performance:

$$I_m = \sum_N \left(x_0(n)^2 + y_0(n)^2 + z_0(n)^2 - (49.5\mu T)^2 \right)^2$$

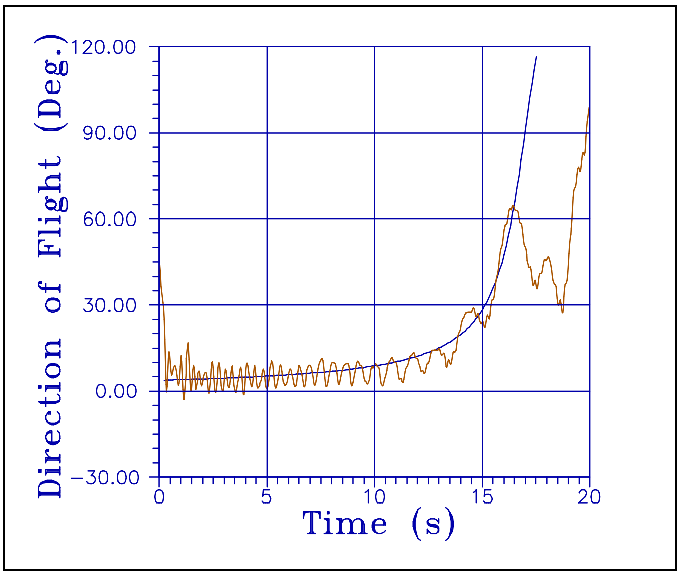
Fortunately, this process converges relatively fast. As an example of this technique, strange fluctuations in the magnetic field magnitude (Skøvnung, 2. flight) were effectively removed in the ballistic flight phase. The original magnetic field is slightly less than the expected value of 49.5 µT, however the value itself is relatively unimportant. Note that the 'near ground' values from the early seconds of flight are larger than the average due to the presence of magnetic particles in the ground.



Magnetic field magnitude from original data and data corrected using the suggested method. Data from Oldenbroek 1992.

At approximately T+17 seconds the rocket entered a flat spin. In this case the rotational energy increases, and undersampling may occur if the sample rate is insufficient. Unfortunately there is no cure for undersampling, which is to be avoided by using a sufficiently high sample rate. Regardless of the sample rate, care should be taken, that the three magnetometer signals are sampled at exactly the same time. If this is not the case, but the inter channel sample delay is known, a post flight digital resampling process may be used for establishing synchronization. This technique has successfully been used on the Skøvnung data, where the analog measurement channels are sampled sequentially at 1/100 second intervals.

When the Z-axis of the magnetometer does not point in exactly the same direction as the length axis of the rocket, and the rocket spins around this axis (roll), there will be some additional oscillations in the values of θ(n). This will mask non-ballistic behavior such as wind influence and stability (pitch) oscillations, and makes the task of finding a best fit more difficult. This is the case for the data recovered from the Oldenbroek flight. The values of θ(n) show an exceptionally vertical trajectory. There must have been some major (wind?) disturbance around T+10 seconds. The 'final' disturbance at approximately T+16 seconds may be due to the errant recovery system. Even though the parachute system did not open, it tried to separate and the rigidity of the rocket has been lost, thereby leaving the rocket unstable.



The best fit values of Ω(n) compared to the measured values of θ(n). Data from the powered and ballistic flight phases, Oldenbroek 1992.

The best fit results from this flight are: Burnout velocity = 166.7 m/s, c_d = 0.41 and the weight of the empty rocket is 9.11 kg, which indicates the expected 5% remanence.

COMPARING THE RESULTS:

As the rocket flown at Oldenbroek was nearly identical to the one flown at Skillingaryd one might ask for an explanation of the differences. First of all, the data from the first flight were much nicer to work with, and even though the best fit was based upon uncompensated data, the value of the burnout velocity is very certain. In practice it turns out, that even in the more complicated Oldenbroek case, no value differing more than about 1 m/s from 166.7 m/s makes an acceptable fit. Furthermore, the difference in burnout velocity is easily explained as aging of the Zinc dust in the Zinc/Sulfur propellant. At Skillingaryd the Zinc dust was about one year old, compared to three months for the Zinc dust used at Oldenbroek. The difference in specific impulse is about 4 seconds (approx 10%), which is within the nature of the Zinc/Sulfur propellant.

The difference in the dragcoefficient estimate is quite remarkable. Even though the finn design was changed, an increase in drag coefficient from 0.3 to 0.41 is unexpected. The truth is that the airdrag in any case is very small, and that the trajectory calculation is rather insensitive to the value of the drag coefficient. A theoretical investigation has shown, that the expected value is to be around 0.39, which the measured values does not contradict. One should note, that oscillations in the flight path increases the drag force and thereby the effective dragcoefficient.

CONCLUSION:

The three-axial magnetometer needs a lot of attention. Special care should be taken, that the sensors are properly aligned, and that all parts nearby the magnetometer are made from nonmagnetic materials such as aluminum, nylon, and stainless steel. However, if this instrument is treated properly, it turns out as a very versatile tool for monitoring flight parameters. The estimates of burnout velocity must be considered as very reliable even in cases where the measurement is jammed by misalignment problems and deviations from the ballistic trajectory caused by the wind. Other parameters estimated with the suggested method might be more sensitive to disturbances, but they do however provide usable information even under heavy conditions. If data from $\theta(n)$ and $\phi(n)$ are combined, all sufficient information is available for creating a flight animation. In the future, the magnetometer might replace the more expensive and power consuming gyroscope, when used together with a quality sampling system and a microprocessor for real time signal processing. Some possible applications are anti-roll systems, intelligent recovery systems, and go/no-go decision making for multistage rockets.

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